High School Trigonometry Curriculum

Course Description: A pre-Calculus course for the college bound student. The term includes a strong emphasis on circular and triangular trigonometric functions, graphs of trigonometric functions and identities and trigonometric equations, polar coordinates, and vectors. This course is primarily taught through lectures, small group activities, and projects dealing with real-life situations. *Graphing calculators are required. See instructor for recommendations.

Scope and Sequence:

Timeframe	Unit	Instructional Topics
10 class periods	Defining Trigonometric Functions and Angles	Topic 1: Angles Topic 2: Foundations Topic 3: Trigonometric Functions
10 class periods	Applying Trigonometric Functions	Topic 1: Cofunctions Topic 2: Evaluate Trigonometric Functions Topic 3: Applications of Trigonometric Functions
11 class periods	Radian Measure	Topic 1: Radians Topic 2: Trigonometric Functions Topic 3: Sectors and Speed
13 class periods	Graphing Trigonometric Functions	Topic 1: Graph Trigonometric Functions Topic 2: Model
13 class periods	Trigonometric Identities	Topic 1: Apply Fundamental Trigonometric Identities Topic 2: Use Identities to Evaluate Trigonometric Expressions

12 class periods	Inverse Trigonometric Functions and Solving Equations	Topic 1: Inverse Trigonometric Functions Topic 2: Solve Trigonometric Equations
11 class periods	Solving Triangles	Topic 1: Law of Sines and Cosines Topic 2: Area

Unit 1: Defining Trigonometric Functions and Angles

Subject: Trigonometry
Grade: 10, 11, 12
Name of Unit: Defining Trigonometric Functions and Angles
Length of Unit: 10 class periods
Overview of Unit: In this unit the student will learn the definitions of the trigonometric functions for angles in standard position on the rectangular plane. The student will review relationship between angles and different ways to express the measurement of an angle.

Priority Standards for unit:
- CBIMVI.2.1.1 Develops and applies the definition of the sine and cosine functions of the degree measure of a general angle in standard position* in relation to the values of the y- and x-coordinates, respectively, of points on the terminal side of the angle.

Supporting Standards for unit:
- MOGeo.SRT.A.2: Use the definition of similarity to decide if figures are similar and to solve problems involving similar figures.
- IMIV.2.1.1 Forms conjectures based on exploring geometric situations with or without technology.
- ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.
- ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
the definition of the sine and cosine functions of the degree measure of a general angle in standard position* in relation to the values of the y- and x-coordinates, respectively, of points on the terminal side of the angle.	Develops	Understand	2
the definition of the sine and cosine functions of the degree measure of a general angle in standard position* in relation to the values of the y- and x-coordinates, respectively, of points on the terminal side of the angle.	Applies	Apply	2

Essential Questions:
1. How do you express angles in different forms?
2. How do you use geometric foundations to find unknown angles?
3. How are trigonometric functions defined and evaluated?

Enduring Understanding/Big Ideas:
1. Angles can be converted between decimal degrees and DMS. An infinite number of coterminal angles can found for any angle in standard position.
2. The properties of parallel lines, triangles and similar triangles can be used to find unknown angles.
3. Trigonometric functions can be defined in the standard x-y coordinate plane. Given a quadrantal angle, or a point on the terminal side of the angle, one can find the value of all six trigonometric functions. This includes recognizing when a trigonometric function is positive or negative.

Unit Vocabulary:

Academic Cross-Curricular Words	Content/Domain Specific
	Angles
	Standard Position
	Coterminal
	Sine
	Cosine
	Tangent
	Cosecant
	Secant
	Cotangent
	Supplementary
	Complementary
	Degrees-Minutes-Seconds
	Quadrantal Angles

Resources for Vocabulary Development: Textbook

Topic 1: Angles

Engaging Experience 1

Title: White Board Activity - Angles

Suggested Length of Time: 25 minutes

Standards Addressed

Supporting:
- IMIV.2.1.1 Forms conjectures based on exploring geometric situations with or without technology.

Detailed Description/Instructions: Teacher will provide a series of problems. Students will work out the problem on individual whiteboards and display so teacher can check for understanding. Questions should cover converting angles and relationships between angles.

Bloom's Levels: Apply

Webb's DOK: 2

Topic 2: Foundations

<u>Engaging Experience 1</u>

Title: Kahoot! Warm Up Activity

Suggested Length of Time: 15 minutes

Standards Addressed

> *Supporting*:
> - MOGeo.SRT.A.2: Use the definition of similarity to decide if figures are similar and to solve problems involving similar figures.
> - ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.

Detailed Description/Instructions: Students will compete in a game of Kahoot! where the questions cover review topics from geometry, including similar triangles, properties of triangles and properties of parallel lines.

Bloom's Levels: Apply

Webb's DOK: 2

Topic 3: Trigonometric Functions

Engaging Experience 1

Title: Question Creation

Suggested Length of Time: 30 minutes

Standards Addressed

Priority:

- CBIMVI.2.1.1 Develops and applies the definition of the sine and cosine functions of the degree measure of a general angle in standard position* in relation to the values of the y- and x-coordinates, respectively, of points on the terminal side of the angle.

Supporting:

- ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

Detailed Description/Instructions: Students will work in small groups to write questions regarding evaluating trigonometric functions in the coordinate plane. Groups will trade their created questions and then find solutions. Groups will discuss solutions with one another.

Bloom's Levels: Apply

Webb's DOK: 2

Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) The students will be assigned the task of creating a unit test and an answer key. The questions should be original and not copied from the textbook. The test should be structured in the following way: Angles: 5 questions

Foundations: 5 questions

Trig Functions: 5 questions

Students will bring their test to class and trade with another student. They will check each other's work using the answer keys they create.

Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Angles	White Board Activity – Angles	Teacher will provide a series of problems. Students will work out the problem on individual whiteboards and display so teacher can check for understanding. Questions should cover converting angles and relationships between angles.	25 minutes
Foundations	Kahoot! Warm Up Activity	Students will compete in a game of Kahoot! where the questions cover review topics from geometry, including similar triangles, properties of triangles and properties of parallel lines.	15 minutes
Trigonometric Functions	Question Creation	Students will work in small groups to write questions regarding evaluating trigonometric functions in the coordinate plane. Groups will trade their created questions and then find solutions. Groups will discuss solutions with one another.	30 minutes

Unit 2: Applying Trigonometric Functions

Subject: Trigonometry
Grade: 10, 11, 12
Name of Unit: Applying Trigonometric Functions
Length of Unit: 10 class periods
Overview of Unit: Students will take an in depth look at the definitions of the six trigonometric functions and how they are evaluated in the standard coordinate plane. Students will evaluate the functions for all special angles and use reference angles to evaluate additional angles. Students will use their calculator to approximate values of trigonometric functions and use these approximations to solve right triangles in real-world scenarios.

Priority Standards for unit:
- IMV.4.1.4 Applies the sine, cosine, and tangent trigonometric ratios to determine lengths and angle measures in right triangles.
- IMV.4.1.3 Develops and applies the properties of 30°-60°-90° and 45°-45°-90° triangles; develops and applies proportional relationships involving the altitude drawn to the hypotenuse of a right triangle.

Supporting Standards for unit:
- IMV.4.1.5 Applies, singly and in combination, the Pythagorean theorem, properties of proportionality, trigonometric ratios, and similarity in solving mathematical and real-world problems.
- ISTE-COMPUTATIONAL THINKER.5.C - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.
- ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
the sine, cosine, and tangent trigonometric ratios to determine lengths and angle measures in right triangles.	Applies	Apply	2
the properties of 30°-60°-90° and 45°-45°-90° triangles; develops and applies proportional relationships involving the altitude drawn to the hypotenuse of a right triangle.	Develops	Understand	2

the properties of 30°-60°-90° and 45°-45°-90° triangles; develops and applies proportional relationships involving the altitude drawn to the hypotenuse of a right triangle.	Applies	Apply	2

Essential Questions:
1. How do you express trigonometric functions in terms of their cofunctions?
2. How do you evaluate trigonometric functions with angles in degree mode?
3. How do you use trigonometric functions to solve real-world problems?

Enduring Understanding/Big Ideas:
1. All six trigonometric functions can be expressed in terms of their cofunction using their cofunction identities. Cofunction identities can also be used to solve for a missing variable in simple trigonometric equations.
2. With angles in degree mode, trigonometric functions can be evaluated using their definitions, special right triangles and reference angles. A calculator can be used to approximate values of trigonometric functions. One can also work backwards from a ratio to find an unknown angle in a simple trigonometric equation.
3. Trigonometric functions can be used to solve right triangles. This can be done from a theoretical lens or in context to many real-world scenarios, including shadows, angle of elevation, angle of depression, bearing and navigation.

Unit Vocabulary:

Academic Cross-Curricular Words	**Content/Domain Specific**
	Sine Cosine Tangent Cosecant Secant Cotangent Angle of Elevation Angle of Depression Bearing Heading Reference Angle Cofunction

Resources for Vocabulary Development: textbook

Topic 1: Cofunctions

Engaging Experience 1

Title: Exploration of Trig Functions of Complementary Angles

Suggested Length of Time: 15 minutes

Standards Addressed

> *Priority*:
> - IMV.4.1.4 Applies the sine, cosine, and tangent trigonometric ratios to determine lengths and angle measures in right triangles.
>
> *Supporting*:
> - ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

Detailed Description/Instructions: Teacher will provide guiding questions that lead students to make a conjecture about complementary angles and cofunctions. Students will work in small groups to develop their ideas. For example, students will compare the values of sin 60°, cos 60°, tan 60°, sin 30°, cos 30°, tan 30° and look for patterns.

Bloom's Levels: Apply

Webb's DOK: 2

Topic 2: Evaluate Trigonometric Functions

Engaging Experience 1

Title: Whiteboard Activity Evaluating Trig Functions

Suggested Length of Time: 25 minutes

Standards Addressed

Priority:

- IMV.4.1.4 Applies the sine, cosine, and tangent trigonometric ratios to determine lengths and angle measures in right triangles.
- IMV.4.1.3 Develops and applies the properties of 30°-60°-90° and 45°-45°-90° triangles; develops and applies proportional relationships involving the altitude drawn to the hypotenuse of a right triangle.

Detailed Description/Instructions: Teacher will provide questions in which students will evaluate the six trigonometric functions. Angles are in degree mode and will include angles in all four quadrants.

Bloom's Levels: Apply

Webb's DOK: 2

Topic 3: Applications of Trigonometric Functions

Engaging Experience 1
Title: Small Group Activity
Suggested Length of Time: 1/2 class period
Standards Addressed

Priority:
- IMV.4.1.4 Applies the sine, cosine, and tangent trigonometric ratios to determine lengths and angle measures in right triangles.

Supporting:
- IMV.4.1.5 Applies, singly and in combination, the Pythagorean theorem, properties of proportionality, trigonometric ratios, and similarity in solving mathematical and real-world problems.
- ISTE-COMPUTATIONAL THINKER.5.C - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

Detailed Description/Instructions: Teacher will provide students with a problem set in which they apply trigonometric functions to real-world situations. Examples of real-world situation would include shadow, navigation, and angles of depression and elevation. Solutions will also be provided in order for students to check their work and ask questions.
Bloom's Levels: Apply
Webb's DOK: 2

Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will write a 1-2-page paper that explains and summarizes the topics that thread throughout the chapter. The following topics should be covered in the paper:
- Deriving Special Right Triangles from a square and equilateral triangle
- Evaluating trig functions in quadrant I
- Evaluating trig functions of the quadrantal angles
- Using reflections to evaluate trig functions in all four quadrants

Students should focus on the connections between these topics as they write their paper.

Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Cofunctions	Exploration of Trig Functions of Complementary Angles	Teacher will provide guiding questions that lead students to make a conjecture about complementary angles and cofunctions. Students will work in small groups to develop their ideas. For example, students will compare the values of sin 60°, cos 60°, tan 60°, sin 30°, cos 30°, tan 30° and look for patterns.	15 minutes
Evaluate Trigonometric Functions	Whiteboard Activity Evaluating Trig Functions	Teacher will provide questions in which students will evaluate the six trigonometric functions. Angles are in degree mode and will include angles in all four quadrants.	25 minutes
Applications of Trigonometric Functions	Small Group Activity	Teacher will provide students with a problem set in which they apply trigonometric functions to real-world situations. Examples of real-world situation would include shadow, navigation, and angles of depression and elevation. Solutions will also be provided in order for students to check their work and ask questions.	½ class period

Unit 3: Radian Measure

Subject: Trigonometry
Grade: 10, 11, 12
Name of Unit: Radian Measure
Length of Unit: 11 class periods
Overview of Unit: This unit will introduce students to radian measure and the Unit Circle. Students will be comfortable converting angles between radian and degree mode, along with evaluating trigonometric functions given radian measure. Students will also learn how to find the arc length and area of a sector. They unit will also introduce the relationship between linear and angular speed through real-world applications.

Priority Standards for unit:
- CBIMVI.2.1.3 Defines the trigonometric functions as functions of the radian measure of a general angle, and describes them as functions of real numbers.
- CBIMVI.2.1.2 Develops radian measure of angles, measures angles in both degrees and radians, and converts between these measures.

Supporting Standards for unit:
- CBIMVI.2.1.4 Develops and applies the values of the trigonometric functions at 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$ radians and their multiples.
- MOGeo.C.B.1: Derive the formula for the length of an arc of a circle.
- MOGeo.C.B.2: Derive the formula for the area of a sector of a circle.
- ISTE-DIGITAL CITIZEN.2.B - engage in positive, safe, legal and ethical behavior when using technology, including social interactions online or when using networked devices.
- ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
_____ the trigonometric functions as functions of the radian measure of a general angle, and describes them as functions of real numbers.	Defines	Understand	2
Defines the trigonometric functions as functions of the radian measure of a general angle, and _____ them as functions of real numbers.	Describes	Understand	3

_____ radian measure of angles, measures angles in both degrees and radians, and converts between these measures.	Develops	Understand	2

Essential Questions:
1. How are radians similar and how are they different from degrees.
2. How can trigonometric functions be applied to angles given in terms of radians?
3. Why are radians sometimes preferred over degrees?

Enduring Understanding/Big Ideas:
1. Radians and degrees are both used to measures angles and can be converted from one form to the other. However, a radian is different from a degree in that it is defined as a length on the unit circle and can be used as simply a number.
2. The unit circle can be used to define trigonometric functions which allows them to be extended to radians and numbers. Given the value of a trig function it is then possible to find the set of angles in radians or degrees that would give that value.
3. The formulas for the area of a sector or the length of an arc of a circle are more simple in radian form. It results in applications to linear and angular velocities being easier in radians.

Unit Vocabulary:

Academic Cross-Curricular Words	**Content/Domain Specific**
Linear Speed Angular Speed	Sine Cosine Tangent Cosecant Secant Cotangent Radians Unit Circle Sector Arc Length

Resources for Vocabulary Development: Textbook

Topic 1: Radians

Engaging Experience 1

Title: Rad War

Suggested Length of Time: 15 minutes

Standards Addressed

> *Priority*:
> - CBIMVI.2.1.2 Develops radian measure of angles, measures angles in both degrees and radians, and converts between these measures.

Detailed Description/Instructions: Teacher will provide students with a deck of cards containing a degree measure on one side of the card and a radian measure on the other side of the card. The students will partner up with a deck of cards, revealing an angle measure. The students will race to convert the angle to the other mode. The winner takes the card.

Bloom's Levels: Understand

Webb's DOK: 2

Topic 2: Trigonometric Functions

Engaging Experience 1

Title: Whiteboards in Small Groups

Suggested Length of Time: 25 minutes

Standards Addressed

Priority:

- CBIMVI.2.1.3 Defines the trigonometric functions as functions of the radian measure of a general angle, and describes them as functions of real numbers.

Supporting:

- CBIMVI.2.1.4 Develops and applies the values of the trigonometric functions at 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$ radians and their multiples.
- ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

Detailed Description/Instructions: Teacher will split class into 3 to 4 groups (5-10 students in each group). One student will create a problem for the group to complete on their whiteboards. The problem should be one where they are evaluating trigonometric functions using special angles in radian mode. The first student to answer correctly then comes up with the next problem for the group. Students should use all six trigonometric functions along with angles from each quadrant. Questions should be discussed within the groups as they arise.

Bloom's Levels: Understand

Webb's DOK: 2

Topic 3: Sectors and Speed

Engaging Experience 1

Title: Quizizz

Suggested Length of Time: 30 minutes

Standards Addressed

Supporting:

- MOGeo.C.B.1: Derive the formula for the length of an arc of a circle.
- MOGeo.C.B.2: Derive the formula for the area of a sector of a circle.
- ISTE-DIGITAL CITIZEN.2.B - engage in positive, safe, legal and ethical behavior when using technology, including social interactions online or when using networked devices.

Detailed Description/Instructions: Teacher will create a Quizizz game via quizizz.com. This is an interactive interface which allows students to work through questions. The students and teachers are able to see the results. The questions should cover sectors and speed.

Bloom's Levels: Analyze

Webb's DOK: 2

Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Design a carousel with two rows of horses. There must 5 feet between the two rows of horses and the maximum linear speed of any child on the carousel is 200 ft/min. Determine the rotation speed and the inner and outer radii of the rows of horses for your carousel (more than one carousel is possible). Present calculations supporting your design.

Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Radians	Rad War	Teacher will provide students with a deck of cards containing a degree measure on one side of the card and a radian measure on the other side of the card. The students will partner up with a deck of cards, revealing an angle measure. The students will race to convert the angle to the other mode. The winner takes the card.	15 minutes
Trigonometric Functions	Whiteboards in Small Groups	Teacher will split class into 3 to 4 groups (5-10 students in each group). One student will create a problem for the group to complete on their whiteboards. The problem should be one where they are evaluating trigonometric functions using special angles in radian mode. The first student to answer correctly then comes up with the next problem for the group. Students should use all six trigonometric functions along with angles from each quadrant. Questions should be discussed within the groups as they arise.	25 minutes
Sectors and Speed	Quizizz	Teacher will create a Quizizz game via quizizz.com. This is an interactive interface which allows students to work through questions. The students and teachers are able to see the results. The questions should cover sectors and speed.	30 minutes

Unit 4: Graphing Trigonometric Functions

Subject: Trigonometry
Grade: 10, 11, 12
Name of Unit: Graphing Trigonometric Functions
Length of Unit: 13 class periods
Overview of Unit: This unit introduces the graphs of the six trigonometric functions. Students will graph the functions and apply transformations to the graphs. Students will be able to state the properties of the functions from both the equations and graphs. The students will write equations to model curves, including real-world applications.

Priority Standards for unit:
- CBIMVI.2.1.5 Constructs the graphs of the trigonometric functions, and describes their behavior, including periodicity, amplitude, zeros, and symmetries.
- MOPA2.2: Use parameter changes to amplitude, period, midline and phase shift to model real-world contexts. Use the form $f(t) = A \sin(B(t+h)) + k$ and explain how to determine each of the parameters A, B, h and k.

Supporting Standards for unit:
- CBIMVI.1.1.1 Determines the domain and range of functions as represented by symbols and graphs, where appropriate.
- CBIMVI.2.2.4 Uses trigonometric functions to model and solve problems in mathematics and other disciplines.
- MOPA2.1: Using a unit circle, create the functions $f(t) = \sin(t)$ and $g(t) = \cos(t)$ to define the position of a point on the circle, at time t. Graph these functions in the Cartesian coordinate plane, and define and explore amplitude, period and midline.
- ISTE-KNOWLEDGE COLLECTOR.3.A - plan and employ effective research strategies to locate information and other resources for their intellectual or creative pursuits.
- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
_____ the graphs of the trigonometric functions, and describes their behavior, including periodicity, amplitude, zeros, and symmetries.	Constructs	Apply	2

Constructs the graphs of the trigonometric functions, and _____ their behavior, including periodicity, amplitude, zeros, and symmetries.	Describes	Analyze	3
_____ parameter changes to amplitude, period, midline and phase shift to model real-world contexts. _____ the form f(t) = A sin(B(t+h)) + k and explain how to determine each of the parameters A, B, h and k.	Use	Apply	3
Use parameter changes to amplitude, period, midline and phase shift to model real-world contexts. Use the form f(t) = A sin(B(t+h)) + k and _____ how to determine each of the parameters A, B, h and k.	Explain	Analyze	3

Essential Questions:
1. How do you graph and apply transformations to the six trigonometric functions?
2. How can trigonometric functions be used to model real-world phenomena?

Enduring Understanding/Big Ideas:
1. The six trigonometric functions can be graphed over multiple periods. The period, amplitude, domain and range can be stated for every function. The graphs can be transformed through changes in amplitude, period, phase shifts, vertical shifts and reflections. The properties of the transformed graphs should also be stated using proper notation.
2. Sinusoidal curves can be used to model phenomena including springs, the rise and fall of tides and weather patterns. Given a sinusoidal curve, an equation can be written in terms of sine and cosine.

Unit Vocabulary:

Academic Cross-Curricular Words	Content/Domain Specific
Period Amplitude	Sine Cosine Tangent Cosecant Secant Cotangent Domain Range Transformations

Resources for Vocabulary Development: Textbook

Topic 1: Graph Trigonometric Functions

Engaging Experience 1

Title: Research Project

Suggested Length of Time: 1 class period

Standards Addressed

 Priority:

- CBIMVI.2.1.5 Constructs the graphs of the trigonometric functions, and describes their behavior, including periodicity, amplitude, zeros, and symmetries.

 Supporting:

- ISTE-KNOWLEDGE COLLECTOR.3.A - plan and employ effective research strategies to locate information and other resources for their intellectual or creative pursuits.

Detailed Description/Instructions: The teacher will provide a packet of questions for students to research on their own. The questions cover the graphs of the six trigonometric functions and also the properties of the parent functions. Students will use textbook and the internet to do the research. Resource located in Schoology.

Bloom's Levels: Apply

Webb's DOK: 2

Topic 2: Model

Engaging Experience 1
Title: Weather Patterns
Suggested Length of Time: 30 minutes
Standards Addressed

Priority:

- MOPA2.2: Use parameter changes to amplitude, period, midline and phase shift to model real-world contexts. Use the form $f(t) = A \sin(B(t+h)) + k$ and explain how to determine each of the parameters A, B, h and k.

Supporting:

- CBIMVI.2.2.4 Uses trigonometric functions to model and solve problems in mathematics and other disciplines.
- ISTE-KNOWLEDGE COLLECTOR.3.A - plan and employ effective research strategies to locate information and other resources for their intellectual or creative pursuits.
- ISTE-COMPUTATIONAL THINKER.5.B - collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.

Detailed Description/Instructions: Each student will partner up with another student. The students will choose a destination, anywhere in the world, to travel. The students will research the average monthly temperatures for the destination and write a sinusoidal equation to model the temperatures. Using their equation, they will determine the expected temperature on their travel dates and summarize their findings. Resource located in Schoology.
Bloom's Levels: Apply
Webb's DOK: 3

Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Sinusoidal curves occur in many different places in the real world. Students will research this and choose a real-world phenomenon that can be modeled with sinusoidal curves. The student will take an in-depth look at this topic. They will collect data points from their specific example and use those to write sinusoidal equations. The students will present their findings to the class. This could be short in-class presentations or short videos the students create.

Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Graph Trigonometric Functions	Research Project	The teacher will provide a packet of questions for students to research on their own. The questions cover the graphs of the six trigonometric functions and also the properties of the parent functions. Students will use textbook and the internet to do the research. Resource located in Schoology.	1 class period
Model	Weather Patterns	Each student will partner up with another student. The students will choose a destination, anywhere in the world, to travel. The students will research the average monthly temperatures for the destination and write a sinusoidal equation to model the temperatures. Using their equation, they will determine the expected temperature on their travel dates and summarize their findings. Resource located in Schoology.	30 minutes

Unit 5: Trigonometric Identities

Subject: Trigonometry
Grade: 10, 11, 12
Name of Unit: Trigonometric Identities
Length of Unit: 13 class periods
Overview of Unit: Students will build on their knowledge of trigonometric identities to develop fundamental, sum and difference, and double- and half-angle identities. They will apply these identities to simplify trigonometric expressions and verify given trigonometric identities. Students will also apply these to evaluate trigonometric expressions given known and unknown angles.

Priority Standards for unit:
- MOPA2.4: Solve problems using trigonometric identities.

Supporting Standards for unit:
- CBIMVI.2.1.7 Develops the fundamental Pythagorean trigonometric identities, sum and difference identities, double-angle identities, and the secant, cosecant, and cotangent functions, and uses them to simplify trigonometric expressions.
- ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
_____ problems using trigonometric identities.	Solve	Evaluate	4

Essential Questions:
1. How can the fundamental identities be used to simplify expressions and verify other trigonometric identities?
2. How can the trigonometric values of given angles be used to generate the trigonometric values of other angles?

Enduring Understanding/Big Ideas:
1. The fundamental identities allow functions to be written in terms of other functions. Then algebraic methods can be applied to simplify expressions or to match it with another expression.

2. Given the trig values for a pair of angles, identities can be used to find the trig values of the sum or difference of the given angles. Given the trig values for an angle, identities can be used to find the trig values for twice and half the angle.

Unit Vocabulary:

Academic Cross-Curricular Words	Content/Domain Specific
Verify	Sine
	Cosine
	Tangent
	Cosecant
	Secant
	Cotangent
	Fundamental Identities
	Reciprocal Identities
	Quotient Identities
	Pythagorean Identities
	Sum and Difference Identities
	Double-Angle Identities
	Half-Angle Identities

Resources for Vocabulary Development: Textbook

Topic 1: Apply Fundamental Trigonometric Identities

Engaging Experience 1

Title: Tutorial Groups

Suggested Length of Time: 45 minutes

Standards Addressed

> *Priority*:
>> ● MOPA2.4: Solve problems using trigonometric identities.

Detailed Description/Instructions: Students will fill out a Tutorial Request Form (TRF) where they choose a problem where they are asked to verify a trigonometric identity. The students will take turns presenting their problems to their group (3-5 students) on a whiteboard. The students will work together to solve the problem by using questioning and discussion. After each problem, the students will summarize their work.

Bloom's Levels: Evaluate

Webb's DOK: 4

Topic 2: Use Identities to Evaluate Trigonometric Expressions

Engaging Experience 1

Title: Formative Online Activity

Suggested Length of Time: 30 minutes

Standards Addressed

> *Priority*:
>
> - MOPA2.4: Solve problems using trigonometric identities.
>
> *Supporting*:
>
> - ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.

Detailed Description/Instructions: Teacher will create a lesson on goformative.com. This website allows students to work out problems on a computer, giving the teacher live results. Students show their work on their screens, giving the teacher the ability to check for understanding across multiple screens. Questions should cover sum, difference, double and half angle identities, both finding exact values and questions in the coordinate plane.

Bloom's Levels: Evaluate

Webb's DOK: 4

Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.)

During the last day of the unit, students will complete Tutorials in small groups (4-6 students). Each student will complete a tutorial request form (TRF) in preparation for the activity. See Schoology for TRF. Students will take turns presenting their questions, with their points of confusion to their group. The groups will use questioning to guide the students to the correct answer. After completing each question, the students will summarize, in writing, the steps taken to solve the problem at hand. After class, the students will complete the back of the TRF on their own.

Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Apply Fundamental Trigonometric Identities	Tutorial Groups	Students will fill out a Tutorial Request Form (TRF) where they choose a problem where they are asked to verify a trigonometric identity. The students will take turns presenting their problems to their group (3-5 students) on a whiteboard. The students will work together to solve the problem by using questioning and discussion. After each problem, the students will summarize their work.	45 minutes
Use Identities to Evaluate Trigonometric Expressions	Formative Online Activity	Teacher will create a lesson on goformative.com. This website allows students to work out problems on a computer, giving the teacher live results. Students show their work on their screens, giving the teacher the ability to check for understanding across multiple screens. Questions should cover sum, difference, double and half angle identities, both finding exact values and questions in the coordinate plane.	30 minutes

Unit 6: Inverse Trigonometric Functions and Solving Equations

Subject: Trigonometry
Grade: 10, 11, 12
Name of Unit: Inverse Trigonometric Functions and Solving Equations
Length of Unit: 12 class periods
Overview of Unit: In this unit the student will evaluate inverse trigonometric functions. They will identify the domain and range of the inverse functions. Students will also solve various forms of trigonometric equations. The students will use trigonometric identities and inverse trigonometric functions to solve equations.

Priority Standards for unit:

- MOPA2.3: Solve equations involving trigonometric functions.

Supporting Standards for unit:

- CBIMVI.2.1.6 Defines and graphs inverses of trigonometric functions with appropriately restricted domains.
- CBIMVI.2.2.3 Solves trigonometric equations, noting the periodic nature of solutions when applicable, and interprets the solutions graphically.
- ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.
- ISTE-DIGITAL CITIZEN.2.B - engage in positive, safe, legal and ethical behavior when using technology, including social interactions online or when using networked devices.
- ISTE-CREATIVE COMMUNICATOR.6.A - choose the appropriate platforms and tools for meeting the desired objectives of their creation or communication.
- ISTE-CREATIVE COMMUNICATOR.6.B - create original works or responsibly repurpose or remix digital resources into new creations.
- ISTE-CREATIVE COMMUNICATOR.6.C - communicate complex ideas clearly and effectively by creating or using a variety of digital objects such as visualizations, models or simulations.
- ISTE-CREATIVE COMMUNICATOR.6.D - publish or present content that customizes the message and medium for their intended audiences.
- ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
equations involving trigonometric functions.	Solve	Apply	4

Essential Questions:
1. How do you evaluate inverse trigonometric functions?
2. How do you find solutions to a variety of trigonometric equations?

Enduring Understanding/Big Ideas:
1. Inverse trigonometric functions can be evaluated using knowledge of special angles and the range rules of inverse trigonometric functions. A calculator can be used to approximate values of inverse trigonometric functions. One should be able to recognize the graphs of the inverse trigonometric functions and pair these graphs with their respective functions.
2. A variety of trigonometric equations can be solved over a given interval with and without a calculator. These include equations in the following forms: linear, quadratic, rational, absolute value, double angle, and half angle. One can also use known trigonometric identities to solve equations. Solutions can be expressed in both degree and radian mode.

Unit Vocabulary:

Academic Cross-Curricular Words	Content/Domain Specific
Interval	Sine Cosine Tangent Cosecant Secant Cotangent Inverse function Range Reference angle Coterminal

Resources for Vocabulary Development: Textbook

Topic 1: Inverse Trigonometric Functions

Engaging Experience 1

Title: Kahoot! Activity

Suggested Length of Time: 20 minutes

Standards Addressed

> *Supporting*:
> - CBIMVI.2.1.6 Defines and graphs inverses of trigonometric functions with appropriately restricted domains.
> - ISTE-EMPOWERED LEARNER1.C - use technology to seek feedback that informs and improves their practice and to demonstrate their learning in a variety of ways.

Detailed Description/Instructions: Teacher will create a quiz on kahoot.com, an interactive online quiz game. The quiz will cover finding exact value of inverse trigonometric functions. The class will complete the quiz as a class, going over points of confusion as they arise.

Bloom's Levels: Apply

Webb's DOK: 2

Topic 2: Solve Trigonometric Equations

Engaging Experience 1

Title: Tutorial Groups

Suggested Length of Time: 45 minutes

Standards Addressed

Priority:
- MOPA2.3: Solve equations involving trigonometric functions.

Supporting:
- ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

Detailed Description/Instructions: Students will fill out a Tutorial Request Form (TRF) where they choose a problem where they are asked to solve a trigonometric equation. The students will take turns presenting their problems to their group (3-5 students) on a whiteboard. The students will work together to solve the problem by using questioning and discussion. After each problem, the students will summarize their work.

Bloom's Levels: Apply

Webb's DOK: 4

Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will create a video or record a screencast where they solve two trigonometric equations and explain their thinking. One equation should be able to be solved without a calculator and the other should be one where a calculator needs to be used. The equations can be original or borrowed from an outside resource. The students should explain their work and thinking thoroughly, including algebraic reasoning, use of inverse trig functions and justifying the quadrants in which solutions are found.

Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Inverse Trigonometric Functions	Kahoot! Activity	Teacher will create a quiz on kahoot.com, an interactive online quiz game. The quiz will cover finding exact value of inverse trigonometric functions. The class will complete the quiz as a class, going over points of confusion as they arise.	20 minutes
Solve Trigonometric Equations	Tutorial Groups	Students will fill out a Tutorial Request Form (TRF) where they choose a problem where they are asked to solve a trigonometric equation. The students will take turns presenting their problems to their group (3-5 students) on a whiteboard. The students will work together to solve the problem by using questioning and discussion. After each problem, the students will summarize their work.	45 minutes

Unit 7: Solving Triangles

Subject: Trigonometry
Grade: 10, 11, 12
Name of Unit: Solving Triangles
Length of Unit: 11 class periods
Overview of Unit: In this unit students will solve triangles using Law of Sines and Cosines, including the ambiguous class of the Law of Sines. Students will apply the Law of Sines and Cosines to real world problems such as navigation. Students will also find areas of triangles when the height is unknown.

Priority Standards for unit:
- MOPA2.5: Solve problems using Law of Sines and Law of Cosines.

Supporting Standards for unit:
- MOGeo.SRT.C.4: Derive the formula A = 1/2 ab sin(C) for the area of a triangle.
- CBIMVI.2.2.4 Uses trigonometric functions to model and solve problems in mathematics and other disciplines.
- ISTE-COMPUTATIONAL THINKER.5.C - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.
- ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

Unwrapped Concepts (Students need to know)	Unwrapped Skills (Students need to be able to do)	Bloom's Taxonomy Levels	Webb's DOK
_____ problems using Law of Sines and Law of Cosines.	Solve	Apply	4

<u>**Essential Questions:**</u>
1. How do you find missing angles and side lengths to all types of triangles?
2. How do you find area of triangles and quadrilaterals?

<u>**Enduring Understanding/Big Ideas:**</u>
1. Triangles can be solved using a variety of methods. Right triangles can be solved using the definitions of the trigonometric functions. The Law of Sines and the Law of Cosines can be used to solve all types of triangles. One is able to recognize when one, two or no triangles exists to fit a given situation. These skills can be applied to real-world situations, including bearing and angle of elevation and depression.

2. The area of a triangle can be found using the formula, $A=\frac{1}{2}ab\sin C$ and also Heron's Formula. The area of a quadrilateral can be found by considering the diagonal that splits the quadrilateral into two triangles.

Unit Vocabulary:

Academic Cross-Curricular Words	Content/Domain Specific
Ambiguous	Sine Cosine Tangent Cosecant Secant Cotangent Law of Sines Law of Cosines Area Bearing Heading Angle of Elevation Angle of Depression

Resources for Vocabulary Development: Textbook

Topic 1: Law of Sines and Cosines

Engaging Experience 1
Title: Solving Triangles on the Whiteboard
Suggested Length of Time: ½ class period
Standards Addressed
 Priority:
- MOPA2.5: Solve problems using Law of Sines and Law of Cosines.

Detailed Description/Instructions: The students will solve a number of triangles using the Law of Sines and Law of Cosines on the white boards. The students will determine which law is needed to solve the triangles.
Bloom's Levels: Apply
Webb's DOK: 4

Engaging Experience 2
Title: Ambiguous Case of Law of Sines
Suggested Length of Time: 30 minutes
Standards Addressed
 Priority:
- MOPA2.5: Solve problems using Law of Sines and Law of Cosines.

 Supporting:
- ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

Detailed Description/Instructions: Students will be placed in small groups. Each group will be given the length of two sides and an angle that form the ambiguous case. The group will then construct both triangles. They will then solve both of their triangles.
Bloom's Levels: Apply
Webb's DOK: 4

Engaging Experience 3
Title: Applications of Law of Sines and Cosines
Suggested Length of Time: ½ class period
Standards Addressed
 Priority:
- MOPA2.5: Solve problems using Law of Sines and Law of Cosines.

 Supporting:
- CBIMVI.2.2.4 Uses trigonometric functions to model and solve problems in mathematics and other disciplines.

- ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.

Detailed Description/Instructions: Students will work in groups of two. They will be given an application problem. They will draw the situation and then solve the problem. The groups of two will be joined to make groups of four. The group will then analyze both solutions.

Bloom's Levels: Apply

Webb's DOK: 4

Topic 2: Area

<u>Engaging Experience 1</u>
Title: Exploring the Area of Quadrilaterals
Suggested Length of Time: 30 minutes
Standards Addressed
> *Priority*:
> - MOPA2.5: Solve problems using Law of Sines and Law of Cosines.
>
> *Supporting*:
> - MOGeo.SRT.C.4: Derive the formula A = 1/2 ab sin(C) for the area of a triangle.
> - ISTE-GLOBAL COLLABORATOR.7.C - contribute constructively to project teams, assuming various roles and responsibilities to work effectively toward a common goal.
> - ISTE-COMPUTATIONAL THINKER.5.C - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

Detailed Description/Instructions: Teacher will provide students with a handout that allows them to explore the area of quadrilaterals. Students will work together in small groups to find all the possible ways to find the area of a rectangle. They will they have to apply these ideas to finding the area of irregular quadrilaterals. Teacher will provide guidance and assistance.
Bloom's Levels: Apply
Webb's DOK: 4

Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) The teacher will create a number of triangles and quadrilaterals with painter's tape in the classroom. These will be taped out on the floor, whiteboards, etc. Each group of students will be given a protractor and ruler or yardstick and assigned a couple of shapes. They will then find the areas using two methods. They will present their findings and methods to the class.

Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Law of Sines and Cosines	Solving Triangles on the Whiteboard	The students will solve a number of triangles using the Law of Sines and Law of Cosines on the white boards. The students will determine which law is needed to solve the triangles.	½ class period
Law of Sines and Cosines	Ambiguous Case of Law of Sines	Students will be placed in small groups. Each group will be given the length of two sides and an angle that form the ambiguous case. The group will then construct both triangles. They will then solve both of their triangles.	30 minutes
Law of Sines and Cosines	Applications of Law of Sines and Cosines	Students will work in groups of two. They will be given an application problem. They will draw the situation and then solve the problem. The groups of two will be joined to make groups of four. The group will then analyze both solutions.	½ class period
Area	Exploring the Area of Quadrilaterals	Teacher will provide students with a handout that allows them to explore the area of quadrilaterals. Students will work together in small groups to find all the possible ways to find the area of a rectangle. They will they have to apply these ideas to finding the area of irregular quadrilaterals. Teacher will provide guidance and assistance.	30 minutes

Unit of Study Terminology

<u>Appendices:</u> All Appendices and supporting material can be found in this course's shell course in the District's Learning Management System.

<u>Assessment Leveling Guide:</u> A tool to use when writing assessments in order to maintain the appropriate level of rigor that matches the standard.

<u>Big Ideas/Enduring Understandings:</u> Foundational understandings teachers want students to be able to discover and state in their own words by the end of the unit of study. These are answers to the essential questions.

<u>Engaging Experience:</u> Each topic is broken into a list of engaging experiences for students. These experiences are aligned to priority and supporting standards, thus stating what students should be able to do. An example of an engaging experience is provided in the description, but a teacher has the autonomy to substitute one of their own that aligns to the level of rigor stated in the standards.

<u>Engaging Scenario:</u> This is a culminating activity in which students are given a role, situation, challenge, audience, and a product or performance is specified. Each unit contains an example of an engaging scenario, but a teacher has the ability to substitute with the same intent in mind.

<u>Essential Questions:</u> Engaging, open-ended questions that teachers can use to engage students in the learning.

<u>Priority Standards:</u> What every student should know and be able to do. These were chosen because of their necessity for success in the next course, the state assessment, and life.

<u>Supporting Standards:</u> Additional standards that support the learning within the unit.

<u>Topic:</u> These are the main teaching points for the unit. Units can have anywhere from one topic to many, depending on the depth of the unit.

<u>Unit of Study:</u> Series of learning experiences/related assessments based on designated priority standards and related supporting standards.

<u>Unit Vocabulary:</u> Words students will encounter within the unit that are essential to understanding. Academic Cross-Curricular words (also called Tier 2 words) are those that can be found in multiple content areas, not just this one. Content/Domain Specific vocabulary words are those found specifically within the content.