Linear Algebra

Course Description:

This course is designed to bridge the gap between high school and college algebra, providing students with a robust foundation in algebra and an introduction to linear algebra. The curriculum covers essential topics to prepare students for advanced studies in mathematics, engineering, computer science, and related fields. Through a combination of theoretical concepts and practical applications, students will gain a deep understanding of algebraic structures and linear systems.

Course outline:

1. **Preliminaries**
    - *Review of basic algebraic operations*
    - *Polynomials and factoring*
    - *Rational expressions and equations*
    - *Exponential and logarithmic functions*
2. **Vectors and Vector Spaces**
    - *Introduction to vectors*
    - *Operations with vectors (addition, scalar multiplication)*
    - *Dot product and cross product*
    - *Vector spaces and subspaces*
    - *Basis and dimension*
3. **Matrices and Matrix Operations**
    - *Definition and types of matrices*
    - *Matrix addition, subtraction, and multiplication*
    - *Determinants and inverses*
    - *Special matrices (identity, diagonal, symmetric, etc.)*
4. **Systems of Linear Equations**
    - *Representation of linear systems*
    - *Gaussian elimination and row reduction*
    - *Homogeneous and non-homogeneous systems*
    - *Applications of linear systems in real-world problems*

5. **Eigenvalues and Eigenvectors**
    - *Definition and properties*
    - *Characteristic equation*
    - *Diagonalization of matrices*
    - *Applications of eigenvalues and eigenvectors*
6. **Orthogonality and Least Squares**
    - *Dot product and orthogonality*
    - *Orthogonal and orthonormal sets*
    - *Projection of vectors*
    - *Least squares approximation and its applications*
7. **Diagonalization and Quadratic Forms**
    - *Diagonalizable matrices*
    - *Process of diagonalization*
    - *Quadratic forms and their properties*
    - *Classification of quadratic forms*
8. **Advanced topics**
    - *Linear transformations*
    - *SVD*
    - *PCA*
    - *Jordan canonical form*

Learning Outcomes:

By the end of this course, students will:

1. Have a solid understanding of foundational algebra concepts.
2. Be proficient in performing vector and matrix operations.
3. Be able to solve systems of linear equations using various methods.
4. Understand the significance of eigenvalues and eigenvectors and their applications.
5. Apply concepts of orthogonality and least squares in practical scenarios.
6. Recognize and utilize advanced topics such as linear transformations, SVD, PCA, and Jordan canonical form in higher-level mathematics and applied fields.

Assessment Methods:

1. Homework assignments
2. Quizzes
3. Midterm and final exams
4. Projects and presentations on advanced topics
5. Participation in class discussions and problem-solving sessions

Instructional Methods:

1. Lectures and interactive discussions
2. Hands-on practice and problem-solving sessions
3. Group projects and collaborative learning
4. Use of mathematical software for simulations and visualizations